Fundamental Physical Constants and the Principle of Parametric Incompleteness

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The principle which allows to construct new physical theories on the basis of classical mechanics by reduction of the number of its axiom without engaging new postulates is formulated. The arising incompleteness of theory manifests itself in terms of theoretically undefinable fundamental physical constants $\hbar$ and c. As an example we built up a parametric generalization of relativistic theory, where the Hubble Law and the dependence of light velocity on time are obtained.

1 INTRODUCTION. QUESTIONS.

It is hard to overestimate the importance of fundamental constants $\hbar$ and c in contemporary physics. They define the structure of the theory's basic formulas so that these formulas can be transformed into corresponding relations of classical physics by fixing limit value of the constants $\hbar = 1/c = 0$. Their numerical value set scale of the phenomena where respective corrections to the classical mechanics become essential. They have become so common that sometimes are "forgotten" when working in the unity system with $\hbar = c = 1$.

Nevertheless there are many questions connected with fundamental constants where full answers are not known yet. Some of them are listed below:

- Why physical theory is uncapable of calculating constants without resorting to the experiment?
- Why fundamental constants emerge in more general physical theories such as quantum and relativistic mechanics but are absent in classical physics?
- Are fundamental constants, which differ from $\hbar$ and c, and relating to them generalization of classical mechanics possible?
- Is the set of possible fundamental constants finite?
- Do physical "constants" depend on time?

There is a great number of constants in physics thus first of all it is necessary to point out what we mean by using the term "fundamental constants".

For instance, speaking about the constant c one usually uses the term *"light velocity"*. At the same time two essentially different constants are used under the same name: the velocity of propagation of electromagnetic waves in vacuum c_{em} and invariant velocity c defining the structure of the relativistic theory. The fact that the value of the electromagnetic waves velocity c_{em} is equal to the value of invariant velocity c is a property of one of the existing interactions while the constant c is build-in any form of substance. In particular, in order to measure the value of invariant velocity there is no need to make electrodynamic experiments. It would be enough to measure a speed of any particle in two frames of reference and get the value from the formula of speed addition: $u' = (u + v)/(1 + uv/c^2)$. Even if photon had a different from zero mass and there were no other massless particles the theory of relativity with the constant c wouldn't have changed.

Consequently, the constant c is a fundamental physical constant, and c_{em} which has the same numerical value is only a parameter of one of the interactions (related to the value of the photon mass) and is not fundamental.

In general, present day's physics consists of three closely related and intersecting parts:

MECHANICS	(classical,quantum,relativity...)	$\hbar$,c,..
INTERACTIONS	(electroweak, strong,...)	e, c_{em}, ..
STRUCTURE	(electron, muon, atom,...)	m_e, m_p,..

MECHANICS sets laws that are applied to any structural unities and relations between them. It is the basis of two other parts of the physics building. For instance, requirement of relativistic invariance and unitarity restricts a class of possible interactions. One and the same INTERACTION can be realized between different STRUCTURAL unities, the variety of which defines diversity of manifistations of our World.

Correspondingly, physical constants can also be divided into three classes (similar classification is given in [1]). Let's hereafter assume to name fundamental only those constants which define formulae structure of theories applicable to all the forms of substance and kinds of interactions, i. e. those constants which define properties of MECHANICS. In that way now we know three fundamental constants: Plank's constant $\hbar$, fundamental velocity c and, apparently, gravitation constant G. Electron charge, masses of elementary particles and other important parameters are not fundamental in the sense stated above.

In classical mechanics fundamental constants are absent. More exactly, their value is trivially fixed (0 or ∞). Gravitation constant G as well as light velocity are present in classical physics but it obtains its fundamental sense only in contemporary theories of space and time.

2 AXIOMATIC BASIS OF THEORIES.

By our opinion the resolution of the questions given in the introduction is connected with the axiomatic analysis of the grounds of physical theories. In mathematics the axiomatic method has been used since the Euclid times but serious attention has been paid to the questions of axiomatic only after the appearance of non-Euclidian geometry and paradoxes in the theory of manifold. The metamathematics with the help of which an axiomatic structure of different parts of mathematics is analised was created by Gilbert and other mathematicians. Any axiomatic system of a theory must possess the following features: independance, incontradictionity and completeness [2].

In particular, completeness means that any statement of a theory must be proved or denied with the help of initial axioms. So classical physics with respect to fundamental constants is complete while the theory of relativity is not because such statements as $c = 300,000\ km/sec$ can neither be proved or denied deductively (of course, without making a corresponding experiment).

For more then two centuries there were attempts to prove the fifth axiom of Euclids parallel geometry. I. e. there was a suspicion that Euclidian axiom system is not independant. Trying to prove the axiom about parallels there were derived many theories which did not depend on that axioma - "perfect geometry" according to Boyai's terminology. A new theory has been created - non-Euclidian geometry. Unlike Euclidian geometry this theory possesses a new constant R-radius of space curvature, the value of which cannot be defined from initial axioms and with $1/R \to 0$ these formulas transfer into respective theorems of Euclidian geometry.

If in the theory there is a constant which value is impossible to derive from initial axioms we shall call it **parametric incompleteness of the theory**. Why does it occur? Obviously because a new, shortened system of axiom contains less information then initial one. A decrease of information results in some incompleteness of the theory's conclusions. This incompleteness can be minimal, i. e. all the functional relations of a theory can be obtained from initial axioms and only final set of constants will remain undefineable.

Following Einstein, there is rather common but axiomatically incorrect way to build up relativistic theory basing on two postulates: the principle of relativity and the principle of constancy of light velocity. It looks like that to build up the theory of relativity it is necessary to add a new axiom of "c_{em}-invariance " to the classical Galilei theory of relativity. It is rather wrong. Lorentz transformations and all the theory of relativity formulas derived from axioms of classical mechanics in which absoluteness of time is denied. And the statement $c = inv$ is a theorem of a theory but is not its initial statement.

The fact that Lorentz transformations can be obtained from simple group considerations without using the second Einstein postulate was known in 1910 yet owing to the works of Ignatowsky, Frank and Rothe ([3],[4] for references). In spite of the fact that this simple and beautiful result has been reprinted in literature several times for the last ninety years it has not become a possession of text-books though. Some latest works of this line can be found in [4] -[10], references for earlier sources in [4].

As it is known there is a definite analogy between geometry and the theory of

relativity. Let's consider a velocity space, i. e. a three-dimensional space,where each point represents one or another inertial system. The vector between two points of this space corresponds to the vector of relative velocity of two systems. The principle of relativity or inertial system equality according to the language of geometry means that the velocity's space is gomogeneous and isotropic, i. e. there are no privileged points (frames of reference) and directions within this space.

If we suppose that the velocity space has a Riemannan structure, we would see that there are *only three* possibilities follow from the principle of relativity: a plane space, a space of negative and positive constant curvature. The first one corresponds to Galilean rule of summing up velocities and to classical mechanics, the second one corresponds to the theory of relativity, the third one, probably, can't be realized in nature though it is formally used in Euclidian theory of field.

The velocity space with constant negative curvature is a Lobachevsky space. The curvature of this space corresponds to the value of fundamental velocity c. In this way, the theory of relativity and classical mechanics are already contained in the principle of relativity and Lorentz transformations can be obtained without using a postulate $c = inv$. In this regard as well as in Euclidian geometry there is a fundamental constant c associated with a decreased number of initial axioms (or information they contain). A giving of the axiom of time absolutivity up makes the theory of relativity a parametrically incomplete.

3 PARAMETRICAL INCOMPLETENESS AS A PRINCIPLE OF ACCORDANCE.

In August, 1900 David Guilbert formulated his famous 23 problems on the II International congress of mathematicians. The sixth problem was a purpose of physics axiomatisation. Almost immediately after that two new mechanics had been constructed: quantum and relativistic. The crucial features of these theories are two fundamental constants $\hbar$ and c.

Within any mechanics one might build up his own axiomatic systems from which all the relations of the theory can be derived. But it is not a satisfactory solution of Guilbert's sixth problem. The existance of several systems of axioms breaks integral structure of physics. Besides, this does not allow to obtain new physical theories by axiomatic means.

After axiomatic definitions of basic concepts (Space, Time, Mass, State) classical mechanics becomes rather formal mathematical theory, the axiomatic system of which must satisfy the conditions of completeness, independence and incontradictivity.

Any decrease of information contained in axiomatic scheme of classical mechanics leads to undefineable (unconcludeable) parametres, functions, etc. There exist such minimal information simplifications of mechanics axioms when only final number of constants becomes undefineable and all the functional relations are derived straightforwardly. Thus, on the basis of classical mechanics it is, probably, possible to form a set of parametrically incomplete theories deductively. In these theories a part of fundamental physical constants will be played by constants which origin is related

to a decrease of the initial information containing in axioms of classical mechanics. At the limit values of these constants we will obtain a classical theory again.

In this way we have, so to say, a converse principle of correspondence. One may obtain classical mechanics not from the quantum theory or the theory of relativity but obtain new fundamental theories from classical physics.

Using the language of algebra, it is necessary to look for all the possible stable deformations of algebraic structures of classical physics. Parameters of deformations inevitably emerging in this connection will be fundamental constants of the new physics. We already know three of such deformations. The construction of other physical theories possessing a corresponding parametric incompleteness looks is more than interesting prospect.

It may happen that despite all the variety (probably infinite) of objects and their interactions in our World there exists a finite number of fundamental physical theories (mechanics) which are parametrically incomplete and can be derived from axioms of classical mechanics.

4 PROJECTIVE THEORY OF RELATIVITY.

As an example of an application of Parametrical Incompleteness Principle let's consider some axioms of classical physics related to the principle of relativity. We are interested in a manifest relations between coordinates x and time t of two observers in different inertial frames S and S':

$$x' = f(x,t), \quad t' = g(x,t) \tag{1}$$

In classical mechanics we requires the following to be realized first of all:

> **Axiom I.** *Transformations of coordinates and time are continuous, differentiable and one-valued functions.*

This requirement is very natural, and though it narrows the class of possible functions of transformation it nevertheless leaves this class rather wide. Each additional axiom reduces arbitrariness of choice of transformations (1) unless this arbitrariness appears parametric or disappear completely (in classical mechanics)

> **Axiom II.** *If some body moves uniformly in the system S it's movement in the system S' will also be uniform.*

The axiom (II) is actually a definition of inertial systems and time. "The time is defined so that moving to be simple." [11].

Despite rather common character of Axiom (I), (II), the functional dependence of transformation can be fixed completely.

$$x' = \frac{Ax + Bt + C}{ax + bt + c}, \quad t' = \frac{Dx + Et + F}{ax + bt + c}, \tag{2}$$

leaving nine parameters $A, B, C, D, E, F, a, b, c$ (see Appendix1)

We should notice that linear-fractional transformations with the same denominator are well known as projective transformations in Geometry. These are more

common transformations when all the straight lines transfer into straight lines [12]. That is an essence of the Axiom (II) in a two-dimensional space (x,t)

Axiom III. *At the moment of time $t = t' = O$ origins of systems coincide: $x = x' = 0$*

Axiom IV. *All the points within the frame of each observer are fixed.*

Axiom V. *If the point of the system S': $x' = 0$ moves at a speed of v relatively S the point of the system S: $x = O$ moves at a speed of $v' = -v$ relatively S'*

The Axiom (III) physically means the possibility of local, simultanuos experiment for both observers which would allow to fix origin of coordinates in space and time.

As far as the point $x' = O$ is stationery in the system according to (IV), it can be derived from the axiom (II) that it moves uniformly and linearly at some speed v relatively S: $x = x_0 + vt$. In this regard (III) the following can be derived: $x_0 = 0$

Analogically the beginning of the system S relatively S' moves at a speed of v' : $x' = -v't'$. In this connection the axiom (V) is introduced. Comparing to previous axioms the fifth axiom is not that "obvious". Moreover, this axiom cannot be realized in absolute (ether) theories where relativistic principle [14] - [16] is violated. We should also point out that it can be considered as a theorem which derives from linearity of transformations (1), isotropy of space and principle of relativity [4]. Let's consider that (V) is an independent axiom allowing the observers in S and S' to coordinate unit of measurement of time(or length) by mutual agreement about the equality of relative speed measured by them.

Taking into consideration Axioms (I)-(V) we can write the following coordinate transformations (1) :

$$x' = \frac{\gamma(v)(x - vt)}{1 + a(v)x - b(v)t}, \quad t' = \frac{\gamma(v)(t - \sigma(v)x)}{1 + a(v)x - b(v)t}, \tag{3}$$

where $\gamma(v), \sigma(v), a(v), b(v)$ - are arbitrary functions of relative speed.

The requirement of relative isotropy of space:

Axiom VI. *At inversion of axles of coordinates of both systems S, S': $x \to -x$ and $x' \to -x'$ of transformations (1) are invariant.*

is valid only in case if $\gamma(v), b(v)$ are even functions and $\sigma(v), a(v)$ -is odd.

The seventh axiom is the key in axiomatic of the relativistic theory It expresses the principle of relativity and equality of inertial systems.

Axiom VII. *There exist at least three equal inertial systems moving with arbitrary speeds.*

If X means the vector (x,t), and $X' = \Lambda(X, v)$ the matrix notation of transformation (3), for all frames S_1, S_2 and S_3 we have: $X_2 = \Lambda(X_1, v_1)$, $X_3 = \Lambda(X_2, v_2) = \Lambda(X_1, v_3)$, so:

$$\Lambda(\Lambda(X_1, v_1), v_2) = \Lambda(X_1, v_3), \quad \forall X_1, v_1, v_2 \tag{4}$$

This functional equation gives the following relations between coefficients of transformation (3):

$$\begin{cases} \gamma_3 = \gamma_1\gamma_2(1+\sigma_1 v_2) \\ \gamma_3 = \gamma_1\gamma_2(1+v_1\sigma_2) \end{cases} \begin{cases} v_3\gamma_3 = \gamma_1\gamma_2(v_1+v_2) \\ \sigma_3\gamma_3 = \gamma_1\gamma_2(\sigma_1+\sigma_2) \end{cases} \begin{cases} a_3 = a_1 + a_2\gamma_1 + b_2\gamma_1\sigma_1 \\ b_3 = b_1 + b_2\gamma_1 + a_2\gamma_1 v_1, \end{cases} \tag{5}$$

where $\gamma_3 = \gamma(v_3)$ etc. The first system of equations in (5) allows us to find a function $\sigma(v)$:

$$\frac{\sigma(v_1)}{v_1} = \frac{\sigma(v_2)}{v_2} = \alpha = const \tag{6}$$

As far as speeds v_1 and v_2 are arbitrary independent values, α is some constant which is fundamental and the same for all the inertial systems. In our World it equals inverse square of "light velocity": $\alpha = 1/c^2$. It is impossible to fix numerical value of this constant without additional axioms. So we have a manifestation of the principle of parametrical incompleteness and appearance of parametrically incomplete generalization of classical mechanics which is known as special theory of relativity.

The requirement of equality of inertial systems together with the axiom (V) leads to the inverse transformation: $X' = \Lambda(X, v) \Rightarrow X = \Lambda(X', -v)$, which results the following (taking into account axiom (VI)):

$$\begin{cases} \gamma^2 = 1/(1 - v\sigma) \\ (\gamma - 1)a = b\gamma\sigma \\ a\gamma v = (1+\gamma)b. \end{cases} \tag{7}$$

The first equation of the system (7) gives a manifest value of Laurence factor γ. The equations of the systems (5),(7) lead to a functional equation (See Appendix 2):

$$\frac{a(v_1)}{v_1\gamma(v_1)} = \frac{a(v_2)}{v_2\gamma(v_2)} = \lambda = const, \tag{8}$$

where λ - is a new fundamental constant which is the same for all the inertial systems. Until now we used only one space dimension. Let's change the axiom (III) for:

> **Axiom III'.** *At the moment of time $t = t' = 0$ of the planes (y, z), (y', z'), which satisfy the equations $x = 0$, $x' = 0$,are parallel. Axles y, y' and z, z' are mutually parallel as well.*

It is easy to obtain formulae for coordinates which are transverse to moving. So we finally have:

$$t' = \frac{\gamma(t - \alpha v x)}{1 + ax - bt}, \quad x' = \frac{\gamma(x - vt)}{1 + ax - bt}, \quad y' = \frac{y}{1 + ax - bt}, \quad z' = \frac{z}{1 + ax - bt}, \tag{9}$$

where

$$\gamma = \frac{1}{\sqrt{1 - \alpha v^2}}, \quad a = \lambda v \gamma, \quad b = \frac{\lambda}{\alpha}(\gamma - 1). \tag{10}$$

These transformations fully correspond to seven axioms formulated above. The only arbitrary quantities are two constants α and λ. As far as these constants are the same for all the inertial reference systems we can call them fundamental.

Obviuosly classical mechanics also follows the axioms (I)-(VII) but it eliminates parametrical incompleteness by introducing two additional statements:

Axiom VIII. *If the speeds of two particles are equal in one frame of reference they will be equal in another one.*

Axiom IX. *If two events are simultaneous in one frame of reference they will be simultaneous in another one.*

The Axiom (VIII) leads to condition $\lambda = 0$, and (IX) - to $\alpha = 0$. The Axioms (I)-(IX) are independent and fully define transformation (1). With their help Galilei equations are derived:

$$x' = x - vt, \quad t' = t. \tag{11}$$

Classical mechanics which contains Axioms (I)-(IX), is parametrically complete theory. There are no undefinable physical constants. We may say that the system (I)-(IX) is informationally complete. If we exclude axiom (IX) in the independent system of axioms (I)-(IX) the quantity of information will be decreased and we shall inevitably obtain some incompleteness of the theory. But this incompleteness is limited only by indefineable constant "c", i.e. it leads to a parametrically incomplete theory. In this respect the axiom (IX) maintains minimum amount of information. The Axiom (VIII) (which omition would lead to generalization of the theory of relativity and new parametrically incomplete mechanics with some new fundamental constant) possesses the same property. We should note that the omition of the second proposal of the axiom (III') would lead to another parametrically incomplete generalization of classical mechanics. If λ and α are non-zero we obtain a theory which generalizes the relativistic theory. It is convenient to call it as **Projective Theory of Relativity**. Obviously, there are possible four limited situations which can be realizad with different scales of observed phenomena. So with $\lambda = 0$ we obtain ordinary Lorentz transformations and with $\alpha \to 0$ - projective generalization of Galilei transformations:

$$t' = \frac{t}{1 + \lambda xv - \lambda tv^2/2}, \quad x' = \frac{x - vt}{1 + \lambda xv - \lambda tv^2/2} \tag{12}$$

As far as λc has a dimension of inverse length any corrections of Lorentz transformations can be detected only long later after initial synchronisating experiment (Axiom III) or at long distances from origin of coordinates.

It seems natural that the decrease of the number of axioms demands a serious reconsideration of our intuitive concept of space and time. First time this happened when a special theory of relativity appeared. We may suppose that relativity of notions can be extended at a further building of parametrically incomplete theories.

5 TRANSFORMATIONS FOR SPEED. LIGHT VELOCITY.

Transformations for speed can be obtained from space-time transformations of (9) by standard way. Defining $u_x = dx/dt$, $u'_x = dx'/dt'$, .. and taking differentials of

(9), we find:

$$u'_x = \frac{u_x - v - (x - u_x t)b/\gamma}{1 - \alpha u_x v + \lambda v(x - u_x t)}, \qquad (13)$$

$$u'_y = \frac{u_y/\gamma + \lambda v(x u_y - y u_x) + (y - u_y t)b/\gamma}{1 - \alpha u_x v + \lambda v(x - u_x t)}. \qquad (14)$$

1.If we consider some point fixed in the system S' ($u'_x = u'_y = 0$), then relatively to observer in S it moves at $t = 0$ with a speed of

$$\vec{u} = \vec{v} + \lambda c^2 \left(1 - \sqrt{1 - \frac{v^2}{c^2}}\right) \vec{r}, \qquad (15)$$

where $\vec{v} = (v, 0, 0)$, $\vec{r} = (x, y, z)$. Despite the fact that relatively to observer in S' all the points of his system are fixed (i. e. they have the same zero speed) S' points have different speed from the point of view of the reference system S. Moreover, they move away in radial directions relative to the point displaced at $\vec{v}\gamma/b$ from the origin of coordinates. The system S' seems as though it expands from the point of view of stationary observer. In this way, relative notion is not only a simultaneousness of events but also a relative motionlessness of objects from the point of view of different observers.

2.We should note that the dependance of formulae (13) on time t doesn't mean non-uniformity of free moving. If particle in the system S moves uniformly and lineary $x = x_0 + u_x t$, a moving in the system S' will be uniform and linear (the system of axioms maintains it). But the speeds u'_x, u'_y depend not only on the speeds u_x, u_y, but also on the location of particles at some fixed moment of time. For example, we have for u'_x:

$$u'_x = \frac{u_x - v - \lambda c^2 x_0 (1 - \sqrt{1 - \alpha v^2})}{1 - \alpha u_x v + \lambda v x_0} \qquad (16)$$

This property as well as relativity of mutual motionlessness is connected with the fact that a projective transformation does not retain parallelism of straight lines.

3.It is easiy to see that if the signal spreads in the system S' at a speed of c, this speed will not be equal to c in the system S'.Moreover, it is dependant on coordinates $\vec{r}, t$. But we can examine that the following value

$$\vec{C}(\vec{r}, t) = \frac{\vec{c} + \lambda c^2 \vec{r}}{1 + \lambda c^2 t} \qquad (17)$$

is an invariant velocity. So for a one-dimension case this value transfers as a velocity (13):

$$C(x', t') = \frac{C(x,t) - v - (x - C(x,t)t)b/\gamma}{1 - \alpha v C(x,t) + \lambda v(x - C(x,t)t)}, \qquad (18)$$

where the same function(17) stands in the left and right part of the firmula. If we take derivative u'_x on u_x and equate it with zero we find that limit can be achieved at $v = c$. The value u'_x with any u_x $v = c$ equals $C(x', t')$. In this way the value defined in (17), can be considered as generalization of light velocity in the theory of relativity.

We should point out that the speed of light is $C(r,t)$ but not c. The constant c is fundamental physical constant and equal $C(r,t)$ only if $r=0$, $t=0$.

For an observer S in $x=0$, light velocity $C(0,t)$ reduces after some time (with $\lambda>0$).We should notice that not so long ago there appeared works in which the idea of dependance of light velocity on time was used for solution of cosmological paradoxes [17]- [22].

We point out again that dependance of light velocity on time and coordinates does not mean non-uniform moving of light signal. Any light signal emitted from some point of space at time $(t_0, \vec{r_0})$, moves with a constant speed $C(\vec{r_0}, t_0)$ along trajectory $\vec{r} = \vec{r_0} + \vec{C}(\vec{r_0}, t_0)(t-t_0)$. In particular, light impulse which passed the origin of coordinates at the moment of synchronising experiment (Axiom III) moves along the trajectory $x=ct$ at a constant speed $C(ct,t)=c$. The dependence of light velocity on time and condition of its constancy along uniform motion trajectory leads to the functional equation:

$$C\left(x_0 + C(x_0,t_0)(t-t_0), t\right) = C(x_0,t_0), \tag{19}$$

which must be true at any x_0, t_0, t. The simplest solution of equation is (17).

6 TRANSFORMATIONS BETWEEN OBSERVERS OF ONE INERTIAL FRAME

In all the formulae of the projective theory of relativity the point $x=0,\ t=0$ is privileged. Actually, linear-fractional transformations are not considered because of their non-homogeneity and consequently non-homogeneity of experiments in space and time. We should notice that priveleged point $x=0,\ t=0$, is obviously associated with an initial synchronising experiment. It is also isolated in space-time in the theory of relativity but it does not result in its non-homogeneity.

Another complication is connected with seeming non-equivalence of observers inside one of the reference system. Let's consider two rest observers which are situated at points $x=0\ \ x=R$. A light signal emitted by the first observer at a speed of $C(0,0)=c$ is received by the second observer in $t=R/c$ time. But he is unable to reflect it with the same speed because in this case it would be back in $t=2R/c$ and would have a speed which would be more than a light velocity at that moment $c > C(0, 2R/c)$.

To solve those problems one must consider transformations between two motionless observers inside one inertial system. In other words, we are looking for generalization of transformations of translation in classical mechanics

$$X = x - R, \quad Y = y; \quad T = t, \tag{20}$$

which would be in accordance with formulae of the projective theory of relativity.

Let's go back to axiomatic method again. As far as we describe transformations between two rest observers $(\vec{X}, T)$ and $(\vec{x}, t)$ in one inertial system, we require axioms (I) and (II) to be fulfilled. Besides, transformation $X = X(x)$ must not be dependant on time (motionlessness of observers) and must be defined by one common parameter-relative distance R which must be chosen the same on agreement

about the unit of measurement of distance. In this way, taking into consideration that $X(R)=0,\ X(0)=-R$ more common linear-fractional transformations have the following form:

$$X=\frac{x-R}{1-\sigma(R)Rx},\quad Y=\frac{\delta(R)y}{1-\sigma(R)Rx},\quad T=\frac{\alpha(R)+\beta(R)x+\gamma(R)t}{1-\sigma(R)Rx},\tag{21}$$

where $\alpha(R),\beta(R),\gamma(R),\delta(R),\sigma(R)$ -are arbitrary functions of the relative distance R. Requiring any x_1,R_1,R_2 complied with the law of composition of transformations: $x_3=X(X(x_1,R_1),R_2)=X(x_1,R_3)$, we obtain:

$$X=\frac{x-R}{1-\sigma Rx},\quad Y=\frac{y\sqrt{1-\sigma R^2}}{1-\sigma Rx}.\tag{22}$$

$$T=\frac{\sqrt{1-\sigma R^2}t+\mu\vec{R}\vec{x}-(1-\sqrt{1-\sigma R^2})\mu/\sigma}{1-\sigma\vec{R}\vec{x}}.\tag{23}$$

where σ, μ are fundamental constants.

It is easy to see that (22) formally coincides with the formula of addition of velocities in the theory of relativity. In three-dimension case transformation has the following form:

$$\vec{X}=\frac{\vec{x}\sqrt{1-\sigma R^2}-\vec{R}+(1-\sqrt{1-\sigma R^2})\vec{R}(\vec{R}\vec{x})/R^2}{1-\sigma\vec{R}\vec{x}}.\tag{24}$$

With $\sigma>0$ this is a Lobachevsky space in Beltrami coordinates of tangent space.

For accordance with the projective theory of relativity we require the light velocity $C(x,t)$ to be invariant for all the observers inside the inertial system. Thus we shall write transformations for speed $U=dX/dT,\ u=dx/dt$

$$U=\frac{u\sqrt{1-\sigma R^2}}{1+\mu Ru-\sigma R(x-ut)}\tag{25}$$

and suppose $U=C(X,T),\ u=C(x,t)$. We obtain a conclusion that invariance is observed if $\mu=\lambda\ \ \sigma=(\lambda c)^2>0$.

In this way we have the following physical situation: homogeneous and isotropic coordinate space inside inertial reference system is a Lobachevsky space of constant negative curvature. At the same time physical vector of direction is a vector tangent to space. Physical length R is connected with geometric s by equation $R=\lambda c\tanh(s)$. Physical time is defined in such a way that free particles move uniformly and lineary for all the rest observers. We should notice that from the formula (23) it can be automatically derived that synchronising procedure looks in the following way: Two observers which are at a relative distance R from each other, for the point lying at an equal distance from them $x=-X=(1-\sqrt{1-(\lambda cR)^2})/(\lambda c)^2R)$, are fixing equal time $T=t$. In case of moving frame of reference it is necessary to use formulae of the projective theory of relativity.

From the geometrical point of view the obtained transformations are six parametric $(\vec{v},\vec{R})$ group transformation leaving forminvariant metric of special kind in a flat Minkowsky space-time (See Appendix 3).

7 HUBBLE'S LAW. EVOLUTION OF THE UNIVERSE.

An interesting conclusion from the formulae of previous sections arises if Doupler's effect is analized within one inertial system.

Let's consider remote **motionless** source of light with coordinates $\vec{R}$ emitting light in the direction to observer which is situated at the beginning of coordinates $x = 0$. The light pulse emitted at the moment of time t_1 according to observer's clock reaches it at the moment t_2. As far as the speed of this signal is constant $C(R, t_1) = C(0, t_2)$ and it moves in the direction towards the observer ($\vec{c} = -c\vec{R}/R$), we have the following relations between R, t_1, t_2:

$$(t_2 - t_1)c = R + \lambda c^2 R t_2. \tag{26}$$

Let's suppose light pulses are emitted with some period $\tau_0 = \Delta T_1$ and are received with a period $\tau = \Delta t_2$. As far as the source's time T and the observer's time t are related according to (23), then at constant $\vec{x} = \vec{R}$, ΔT equals $\Delta t/\sqrt{1 - (\lambda c R)^2}$. Thus the period of emission is $\tau_0 = \Delta t_1/\sqrt{1 - (\lambda c R)^2}$, and having entered cosmological parameter of redshift we finally obtain:

$$1 + z = \frac{\tau}{\tau_0} = \sqrt{\frac{1 + \lambda c R}{1 - \lambda c R}}. \tag{27}$$

Interpreting the redshift according to Doupler's formula we obtain Hubble's law:

$$\vec{V} = \lambda c^2 \vec{R} \tag{28}$$

As we saw above the privelegness of the point $x = 0,\ t = 0$ is connected with the initial synchronising experiment for concording the units of length and time by different observers. Nevertheless, space is homogeneious and isotropic and therefore all the observers should be equivalent.

But homogeneity in time is not quite usual. First of all it's usualness is associated with the dependence of local light velocity (or maximum possible physical velocity) on time:

$$C(0, t) = \frac{c}{1 + \lambda c^2 t}. \tag{29}$$

In the past light velocity turned into infinity with $t_0 = -1/(\lambda c^2)$. The fact that the "beginning of Time" t_0 is away from time $t = 0$, by the value proportional to fundamental constants, is also associated with the procedure of defining units of measurement.

You may find the explanation of it in such example. Suppose, that some observer at the moment of time $t = 0$ had defined the unit of length (ruler), the unit of time (sec) and measured light velocity and received the result $c = 300000\ km/sec$. In some period of time T his distant descendants found the unit of length but they do not know the unit of time. But it is known from "ancient manuscripts" that light passes 300000 km per second. Descendants define respectively their clocks and light velocity measured by them from this moment equals:

$$C(0, T) = \frac{c}{1 + \lambda c^2 T} \tag{30}$$

There is used the same *numerical value* of fundamental constant c in the formulae (29),(30). But time t and time T, are evidently different. It is easy to find valation between of times $T = T(t)$. As far as $dx = C(T)dT = C(t)dt$, and $T(\tau) = 0$, we obtain by integrating the formula:

$$T = \frac{t - \tau}{1 + \lambda c^2 \tau}, \tag{31}$$

generalizing the notion of replacement of time. For descenders as well as for their ancestors "the beginning of Time" is distant for the same value $(\lambda c^2)^{-1}$, measured in different units of time.

Now let's discuss applicability of theory consrtucted herein to the Real World. As far as Hubble's effect is naturally described within the projective theory of relativity it would be natural to associate Hubble's constant $H = 65\ km/sec/Mps = 6.7\ 10^{-11}\ year^{-1}$ with the constant λc^2. In this case the change of light velocity in time would be the following:

$$\frac{\Delta C}{C} = \lambda c^2 t = -6.7\ 10^{-11} \frac{t}{year}. \tag{32}$$

In this way we have the following cosmological model. We live in stationary space of constant negative curvature $(\lambda c)^{-1} = const$. The evolution of our World is connected not with the expansion of Universe but with the variability of speed of light $C(r, t)$ with time and distance. This leads to the observed redshift for radiation of distant objects.

There is also another and not so radical possibility of interpretation of Hubble's extension. If $\lambda c^2 \ll H$, our theory gives only small corrections to Hubble's law, when traditional extension contributes more within the limits of the theory of Big Bang. In this case the dependence of light velocity on time $C(x, t)$ can be considerable only within little times from the beginning of Big Bang.

8 CONCLUSION. QUESTIONS.

So taking proposed relativistic theory as an example, we shown that there is a possibility to obtain new physical theories generalizing classical mechanics by reducing its axiomatic base. If the system of axioms is independent we find incompleteness of the theory which can be minimal in some cases and can cause the emerging of fundamental physical constants and respective theories. Thus all the generalizations of classical mechanics are parametrically incomplete theories, and fundamental constants are manifestations of this incompleteness.

Quantum theory with Plank's constant is also a parametrically incomplete theory. The elements of its axiomatic building can be found at Dirac. It is shown in [23] that Plank's constant emerges from natural classical requirement if to exclude the axiom of physical values commutation. We should notice that one of the axiomatic directions of quantum mechanics - a quantum logic, also arises from the idea of decrease of number of axioms. In this case we consider the system of axioms of Boolean logic, where the axiom of distributive [24] is omitted. The resulted nondistributive lattice happens to be isomorphic for some quantum-mechanical systems.

Deductive, preexperimental path of building new theories looks very attractive. In this regard the following questions arise:

- How to formalize the concept of information contained in physical axiom?
- Is there an efficient procedure of search of axioms, which being omitted cause minimal informational lose-parametrical incompleteness?
- Are parametrical generalizations of quantum mechanics possible?
- Is the amount of information contained in the system of axioms of classical mechanics, and, therefore, a number of its possible generalizations and fundamental constants, limited?
- Is the Nature limited by parametrical incompleteness?

Of course, the list of these questions can be continued. Anyway, the investigation of axiomatic of physics is not only academically interesting for it extenses our understanding of Nature, but also it can lead us to constructive results allowing experimental testing.

APPENDIX 1.

Let's consider arbitrary, independant, differentiating transformations of the coordinate x and time t:

$$x' = f(x,t), \quad t' = g(x,t). \tag{33}$$

We require the system of coordinates (x,t) and $(x't')$ to satisfy the definition of inertial reference systems:

$$\frac{du}{dt} = 0 \quad \Longrightarrow \quad \frac{du'}{dt'} = 0, \tag{34}$$

i.e. if a moving of a body is uniform in one system, it would be uniform in another one.

According to definition the speeds in each system are $u = dx/dt$ and $u' = dx'/dt'$, thus:

$$u' = \frac{f_x u + f_t}{g_x u + g_t}, \tag{35}$$

where $f_x = \partial f(x,t)/\partial x$, etc. Differentiating (35) on $dt' = (g_x u + g_t)dt$ and equating coefficients of speed u (taking into consideration its arbitrariness) to be zero, we obtain the system of differential equations:

$$\begin{aligned}
f_{xx} g_x &= g_{xx} f_x & (36)\\
f_{tt}\ g_t &= g_{tt}\, f_t & (37)\\
f_{xx} g_t + 2 f_{xt} g_x &= g_{xx} f_t + 2 g_{xt} f_x & (38)\\
f_{tt} g_x + 2 f_{xt} g_t &= g_{tt} f_x + 2 g_{xt} f_t. & (39)
\end{aligned}$$

Let's introduce Jacobian of transformations different from zero (33) $D = f_x g_t - f_t g_x$. Taking its derivatives by on x and t with the help of (36) - (39) we obtain equations:

$$2\frac{D_x}{D} = 3\frac{f_{xx}}{f_x} = 3\frac{g_{xx}}{g_x}; \qquad 2\frac{D_t}{D} = 3\frac{f_{tt}}{f_t} = 3\frac{g_{tt}}{g_t}, \tag{40}$$

which are easily integrated and give the following equations:

$$\frac{f_t}{f_x}=\frac{A(x)}{B(t)};\qquad \frac{g_t}{g_x}=\frac{\bar{A}(x)}{\bar{B}(t)};\qquad \frac{g_t}{f_t}=\frac{\bar{A}(x)}{A(x)};\qquad \frac{g_x}{f_x}=\frac{\bar{B}(t)}{B(t)}, \tag{41}$$

where $A(x), B(t), \bar{A}(x), \bar{B}(t)$ - are arbitrary functions. We should notice that two last equations in (41) are direct results of (36),(37).

Let's multiple the equation (38) by f_t, and (39) by $-f_x$ and add. Then with the help of (36), (37) we obtain differential equation only for the function $f(x,t)$:

$$f_{xx}f_t^2+f_{tt}f_x^2=2f_{xt}f_xf_t. \tag{42}$$

Taking f_{xx} and f_{tt} from the first equation (41) and putting it in (42), we obtain:

$$A'(x)=-\dot{B}(t)=\alpha. \tag{43}$$

As far as t and x are independent arguments, α is an arbitrary constant. The equations(36)-(39) are symmetric under replacement of f for g, thus we have similar equations for $\bar{A}(x)$ and $\bar{B}(t)$. Thus:

$$\begin{aligned} A(x) &= \alpha x+\beta; \qquad B(t)=-(\alpha t+\gamma);\\ \bar{A}(x) &= \bar{\alpha}x+\bar{\beta}; \qquad \bar{B}(t)=-(\bar{\alpha}t+\bar{\gamma}), \end{aligned} \tag{44}$$

where $\alpha,\beta,\gamma,\bar{\alpha},\bar{\beta},\bar{\gamma}$ - are constants which do not depend on x and t.

Integrating the third and fourth equations (41), we have:

$$g(x,t)=\frac{\bar{A}(x)}{A(x)}f(x,t)+M(x)=\frac{\bar{B}(t)}{B(t)}f(x,t)+N(t) \tag{45}$$

or

$$f(x,t)=(M(x)-N(t))/\left(\frac{\bar{B}(t)}{B(t)}-\frac{\bar{A}(x)}{A(x)}\right), \tag{46}$$

where $M(x), N(t)$ - are arbitrary functions. Putting (46) in the first equation (41), we have:

$$(\alpha x+\beta)M'(x)+\alpha M(x)=(\alpha t+\gamma)\dot{N}(t)+\alpha N(t)=\sigma, \tag{47}$$

where σ - is arbitrary constant. The equations (47) are easily integrated and give for $M(x)$ and $N(t)$:

$$M(x)=\frac{\sigma x+\lambda}{\alpha x+\beta};\qquad N(t)=\frac{\sigma t+\mu}{\alpha x+\gamma}, \tag{48}$$

which along with (41) finally lead us to lineary-fractional transformations with the same denominator.

APPENDIX 2.

From the second equation of the system (7) and first two systems (5) we have:

$$\frac{b_3}{a_3}=\frac{\gamma_3-1}{\alpha\gamma_3 v_3}=\frac{\gamma_1\gamma_2(1+\alpha v_1v_2)-1}{\alpha\gamma_1\gamma_2(1+\alpha v_1 v_2)} \tag{49}$$

Let's divide the equations of the third system in (5) by each other and introduce designation: $F = a/(\alpha v \gamma)$. Then:

$$\frac{F_1(\gamma_1 - 1)\gamma_2 + F_2\gamma_1\gamma_2(\gamma_2 - 1 + \alpha v_1 v_2 \gamma_2)}{F_1 v_1 + F_2(\gamma_2(v_1 + v_2) - v_1)} = \frac{\gamma_1\gamma_2(1 + \alpha v_1 v_2) - 1}{v_1 + v_2}, \tag{50}$$

where

$$F_1(v_1\gamma_1 + v_2\gamma_2 - \gamma_1\gamma_2(v_1 + v_2)) = F_2(v_1\gamma_1 + v_2\gamma_2 - \gamma_1\gamma_2(v_1 + v_2)) \tag{51}$$

or $F_1 = F_2$

APPENDIX 3.

It is easy to make sure that formulae of the projective theory of relativity have the following invariants:

$$\frac{c^2t^2 - x^2}{(1 + \lambda c^2 t)^2} = inv, \quad \frac{c^2 t_1 t_2 - x_1 x_2}{(1 + \lambda c^2 t_1)(1 + \lambda c^2 t_2)} = inv. \tag{52}$$

Using (52) it is easy to obtain forminvariant metric at projective transformations:

$$ds^2 = \frac{1 - (\lambda c \vec{x})^2}{(1 + \lambda c^2 t)^4} c^2 dt^2 + \frac{2\lambda c^2 \vec{x} d\vec{x} dt}{(1 + \lambda c^2 t)^3} - \frac{d\vec{x}^2}{(1 + \lambda c^2 t)^2}. \tag{53}$$

We can also make sure that metric (53) remains forminvariant respective transformations inside one reference system. In this way we have six-parametrical group of transformations $(\vec{v}, \vec{R})$.

Metric (53)can be written in the following way:

$$ds^2 = -\frac{1}{(1 + \lambda c^2 t)^2} \left(d\vec{x} - \frac{\vec{c} + \lambda c^2 \vec{x}}{1 + \lambda c^2 t} dt \right) \left(d\vec{x} - \frac{-\vec{c} + \lambda c^2 \vec{x}}{1 + \lambda c^2 t} dt \right). \tag{54}$$

As far as $ds^2 = 0$ for spreading of light, we obtain the equation for light velocity again $d\vec{x}/dt = \vec{C}(\vec{x}, t)$.

By introducing new time

$$\tau = \tau_0 - \frac{1}{\lambda c^2} \frac{\sqrt{1 - (\lambda c \vec{x})^2}}{1 + \lambda c^2 t}, \tag{55}$$

we obtain Robertson-Walker's metric:

$$ds^2 = c^2 d\tau^2 - a(\tau)^2 (d\chi^2 + \sinh^2\chi \;\; d\Omega^2), \tag{56}$$

where $a(\tau) = c|\tau - \tau_0|$ and spherical system of coordinates with radius $\lambda c |\vec{x}| = \tanh \chi$ is introduced. With the help of well-known transformations [11] we may transfer metric (56) into Minkowsky metric.

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